

Write your name on the cover of the test booklet and nowhere else. Enclose this sheet with the booklet. Failure to follow these directions will cost you 1 point. The test has 100 points (to be scaled up to 170 points) and is scheduled to take 50 minutes. Therefore, expect to spend 1 minute for every 2 points. For example, a 12-point question should take 6 minutes. I cannot give much extra time.

**Show all work for all questions.**

1) (2 points) What is the better known name of  $\begin{bmatrix} 0 & \vdots & (\nabla f)^T \\ \dots & \dots & \dots \\ \nabla f & \vdots & \nabla^2 f \end{bmatrix}$ ?

2) (4 points) Prove that in general  $|\bar{H}_1| < 0$ .

3) (10 points) Prove that if  $Y = F(X_1, X_2)$  is convex, then  $F_{ii}'' \geq 0$  for all  $i$ .

4) (12 points) Find the degree of homogeneity for the function  $Q(L, K) = 6L^{1/3}K^{1/2}$  using the formal proof. What does this tell you about the returns to scale of producing this item?

5) (16 points) For the equations  $y = 3x^2 + 2xy + y^2 + 7$ , find the first order total differentiation ( $dy$ ) and the second order total differentiation ( $d^2y$ ). What information does this give about the concavity of the function.

6) (16 points) If  $f(x, y, z) = x^2 + 3xyz + 4y^2$ , then find  $\nabla f$ ,  $\mathbf{H}$ , and  $\mathbf{H}$ .

7) (20 points) Find the bordered Hessian for  $f(x, y) = 5xy$ . Determine if  $f$  is quasi-concave, quasi-convex, or neither.

8) (20 points) Suppose  $U(C, A) = 6A^{1/2}C^{1/3} + 6A^{1/3}C^{1/2}$ . Find the slope of the indifference curve at the point (1, 64). Note that  $C$  is on the  $x$ -axis. Given your answer, how many apples would this person be willing to trade for a carrot? Briefly state how you reached that conclusion.