

This review sheet is intended to cover everything that could be on the exam; however, it is possible that I will have accidentally left something off. You are still responsible for everything in the chapters covered except anything that I explicitly say you are not responsible for. Therefore, if I left something off of this sheet, it can still be on the exam. There will be no multiple-choice questions. Most of the questions will be like the ones in the homework assignments, and possibly a few definition questions, but I am more likely to ask questions that make you use the definitions rather than recite them. I will probably ask one of the questions from the book at the end of the chapters.

The review session will be at a time to be determined, probably Thurs. 11/8 in the normal room.

Section 8.1: What is a **matrix**? Note, we will not be dealing with game matrices. Be able to create a **Leontief Input-Output Matrix**. Remember the columns must total to less than 1 because they are the dollar amount of inputs used to produce \$1 worth of goods. Know what the following terms mean: **column matrix, column vector, row matrix, row vector, square matrix, diagonal matrix, identity matrix, and null matrix**. (That last one is stupid.) Matrices are **equal** if every entry is the same.

Section 8.2: Know how to **add** and **subtract** matrices, do **scalar multiplication** and **matrix multiplication**. Note that if A is $m \times n$ then in order to multiply AB , then B must be $n \times k$ and the product AB will be $m \times k$. (It is possible for m to be the same as n and/or k .) *When multiplying matrices, go across the rows of the first matrix and down the columns of the second and add the products. The answer goes in the row you went across and in the column you went down.* (I suspect at this point you have done it so much, you won't get confused.) Remember, that **premultiplying** will not normally give you the same answer as **postmultiplying** partly because it is possible only one of them can be done. (Note, that $IA=AI$ and $AA^{-1}=A^{-1}A$.) The **migration model** is just $x_i = P^i x_0$. (I changed the superscripts to subscripts so they do not get confused with the exponent on the P .)

Section 8.3: The **transpose** of a matrix is A^T is just rows becoming columns and vice versa. A **symmetric matrix** has $A^T=A$. For all matrices, $(A^T)^T = A$ and $(A+B)^T = A^T+B^T$. If both products are defined then $(AB)^T = B^T A^T$.

Section 8.4: An **idempotent matrix** is one where $A = A^2$. A **partitioned matrix** is one with partitions in it. *It works like a regular matrix for addition etc., providing that the dimensions are the same and the dimensions of the partitions are also the same.* The **trace of a matrix** is the sum of the diagonal elements. *The $trace(AB) = trace(BA)$ providing both AB and BA are the defined, even if AB is a different dimension from BA .*

Section 9.1: An **inverse matrix** is one where $A^{-1}A=I=AA^{-1}$. If A^{-1} does not exist, then A is called **singular** otherwise it is **non-singular**. The **determinant of a matrix** $|A|$ for a 2×2 matrix is just $a_{11} a_{22} - a_{12} a_{21}$. *If A is 2×2 , A^{-1} is gotten by taking A , swapping the main diagonal matrix and changing the signs off-diagonal elements and then dividing by the determinant.* (Note the

description on Pages 305-306 is actually the minor, cofactor, adjoint method from Section 9.3.) Note that $|A| = |A^T|$, if B is A with two rows swapped or two columns swapped, then $|A| = -|B|$ even for larger matrices. Theorems 9.3 - 9.4 can be summarized, “if the rows or columns are linearly dependant, then the determinant is zero.” If you add a multiple of one row (column) to another, then the determinant does not change values. The determinant of a **triangular matrix, both upper triangular and lower triangular**, is the product of the diagonal. If you multiply a row by a scalar λ , then you have multiplied the determinant by λ too. If A and B are square and the same dimensions, then $|AB| = |A||B|$. If you are solving the equation $Ax=b$, then $x=A^{-1}b$. Do not worry about the geometric interpretation of the determinant, although it is twice the area between the two vectors.

Sections 9.2 and 9.3: The **minor** M_{ij} is the determinant of A after you get rid of row i and column j. The **cofactor** $C_{ij} = (-1)^{i+j} M_{ij}$. You can do the **cofactor expansion** by doing:

$|A| = \sum_{i=1}^n a_{ij} C_{ij}$ or $|A| = \sum_{j=1}^n a_{ij} C_{ij}$ *In other words, you can choose any row or any column and expand by it. That means multiplying every element in the row or column by its corresponding cofactor. The **matrix of minors** is just a matrix where each entry is the minors of A.*

The **adjoint** of the matrix A is just the transpose of the **matrix of the cofactors**. Therefore, we get $A^{-1} = \text{adj}(A)/|A|$. Other important facts are $|A| = 1/|A^{-1}|$, $(AB)^{-1} = B^{-1}A^{-1}$ providing A^{-1} and B^{-1} exist. $(A^{-1})^{-1} = A$. If A is a diagonal matrix, then every entry in A^{-1} equals $1/a_{ii}$ where a_{ii} are the corresponding entries in the original matrix A.

Section 9.4: **Cramer’s Rule** is simple. If you have a matrix equation $Ax=b$, then $x_i = |A_i|/|A|$ where A_i is A with column i replaced by the vector b. Note that **Open Leontief Input-Output Matrix** is one case where Cramer’s Rule can be used (but ironically the book does not) providing that you realize the equation is really $(I-A)x=b$. *Note, that unless there are strange values for A (like a column of A adding to more than 1), then $(I-A)^{-1}$ will only have positive values.* The **Closed Leontief Input Output Matrix** is done just like the open one except then you multiply the transposes of the two input vectors, e^T (employees) and k^T (capital) by your final x to find out how much labor and capital you need to produce what is desired. Then check to see if you have enough. Do not worry about what to do if you do not have enough labor or capital. To do that well you need to maximize output subject to constraints and that is taught in ECON 477.

Section 11.1: **Partial derivatives** are basically the same as regular derivatives except there are more than one independent variable. So, for example, $w=f(x,y,z)$. You treat the other variables as constants. To signify partial derivative, you use ∂ instead of d . If you want to take the derivative using the ' notation, you must put a subscript. For example $\partial f/\partial x \equiv f'_1 \equiv f'_x$. *The interpretation of the partial derivative is that is the slope in the one direction. (See the diagrams on page 395.)* An **additively separate** function is one where all terms have only one of the independent variables. Note that the marginal product function (and really any marginal function) is the derivative with respect to that input. For example, $MPL = \partial TP/\partial L$ and $MPK = \partial TP/\partial K$. A **Cobb-Douglas** function is of the form $y = a * x_1^{\alpha_1} x_2^{\alpha_2} \dots x_n^{\alpha_n}$. The **constant elasticity of substitution (CES)** is of the form $y = a[\delta x_1^{-r} + (1 - \delta)x_2^{-r}]^{-1/r}$. If f is a function of

x and y which in turn are functions of t, $f(x(t), y(t))$, then $\frac{df}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt}$. Basically that is the chain rule from earlier in the semester (the derivative of $f(g(x))$) but written slightly different because of the partial derivatives and full derivatives.

Non-graded Assignment #8A to be reviewed with Assignment #8.

1) (10 points) Use Cramer's Rule to prove that if you have a system of equations such that all of the equations can be written as $a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n = 0$, then the solution is the trivial solution.

2A) (30 points) Suppose that \$1 worth of farm products (f) uses \$.2 worth of farm products, \$.1 of machinery (m) and \$.3 worth of energy (e). \$1 of machinery uses \$.3 of machinery and \$.1 of energy. \$1 of energy uses \$.2 of machinery and \$.2 of energy. Set up the open Leontief Input-Output Matrix. If there is demand for \$100 of farm products, \$30 of machinery, \$40 worth of energy, then how much of each must be made?

2B) (10 points) If \$1 of farm products uses 5 units of labor, \$1 of machines uses 3 units of labor, and \$1 of energy uses 1 unit of labor, then how much labor is needed? Make sure you set it up in matrix format before you give me the answer.

3) (10 points each) Suppose $f(x, y, z) = 3x^2y - 5x + 4xyz^3 + y/z$. Find each of the following:

A) $\partial f / \partial x$

B) f'_y

C) f'_z

4) (20 points) Suppose the total output is $Q = 12K^{1/3}L^{1/4}$. Find the MPK and MPL.