This review sheet is intended to cover everything that could be on the exam; however, it is possible that I will have accidentally left something off. You are still responsible for everything in the chapters covered except anything that I explicitly say you are not responsible for. Therefore, if I left something off of this sheet, it can still be on the exam. There will be no multiple-choice questions. Most of the questions will be like the ones in the homework assignments, and possibly a few definition questions, but I am more likely to ask questions that make you use the definitions rather than recite them. I will probably ask one of the questions from the book at the end of the chapters.

The review session will be at a time to be determined, probably Thurs. 12/6 in the normal room.

Section 11.2: **Second-order partial derivatives** are basically the first-order partial derivatives again except that you can now have **cross-partial derivatives** which is where the first derivative is with respect to one variable and the second is with respect to a second variable. The **gradient vector ∇f** is the vector of the first partial derivatives i.e., $[f_1' \ f_2' \dots f_n']^T$ (Note the transpose.) The **Hessian matrix** is signified by **H** or ∇^2 or as your book does it ∇_2 is the matrix of all n^2 second-order partial derivatives where element a_{ij} is f_{ij}''. **Young's theorem** says that $f_{ij}'' = f_{ji}''$ if all first and second derivatives are continuous (which is the case in 99.9% of economics). Note that if f(**x**) is additively separate, the **H** is diagonal.

11.3: The **first-order total derivative** is where you use the normal d and you do all derivatives. For example, if $y = f(x_1, x_2, x_3)$ then $dy = f_1'*dx_1 + f_2'*dx_2 + f_3'*dx_3$. If you have F(x, y) = 0 then $dy/dx = -F_x'/F_y'$. That is called **implicit differentiation** and uses the **implicit function theorem**. It can be extended to multiple variables. **Level curves, a.k.a., isobars** are lines where the function equals a constant. The most common ones in economics are **indifference curves** and **isoquants**. The slope of them can be found by subtracting the constant from F(x, y) = c from both sides and then using the implicit function theorem. The $\mathbf{MRTS_{LK}}$ is the negative of the slope of the isoquant = -ΔK/ΔL = MPL/MPK. Similarly, the $\mathbf{MRS_{XY}}$ = $-\Delta Y/\Delta X = MU_X/MU_Y$. Notice that in both cases the equations look upside down. Note that a **positive monotonic transformation T** of a function F will yield the same isobar map. The transformation must have T(F)>0 and T'>0. We can use that to show all Cobb-Douglas functions which have the same ratio of a to b, will have the same isobar map, thus will result in the same optimal point. Let $\tilde{u} = (1/A^k)*U^k$ where $k = 1/(a+b)$ and $U(x_1, x_2)=Ax_1^a x_2^b$.

11.4: The **second-order total differentiation** is $d^2y = f_{11}''dx_1^2 + 2f_{12}''dx_1dx_2 + f_{22}''dx_2^2$. Hint, you are just taking every derivative twice and multiplying by d whatever you took the derivative with respect to. The 2 in front of the second term is because $f_{12}'' = f_{21}''$. If $d^2y > 0$ when at least one of dx_1 or dx_2 is not zero, then it is strictly convex. *The easiest way to think about this is that in two dimensions $y=x^2$ is convex and $y''>0$.* Similarly If $d^2y < 0$ when at least one of dx_1 or dx_2 is not zero, then it is strictly concave. *If you change the > and < in this paragraph to ≥ and ≤ respectively, you eliminate the word strictly.* The easiest way to do these tests is to use the theorem on Page 443, if **H** is **positive definite** then f(**x**) is strictly convex, if **H** is **negative definite** then f(**x**) is strictly concave, if **H** is **positive semi-definite** then f(**x**) is convex, and if **H** is **negative semi-definite** then f(**x**) is concave. Unfortunately, this requires a little bit of Section 10.3 which we skipped. Basically if $|H_i| > 0$ for all i, **H** is positive definite. If $|H_i| \geq 0$ for all i, **H** is positive semi-definite. If $|H_i| <0$ for odd i, and $|H_i| >0$ for even i, then **H** is negative definite. If $|H_i| \leq 0$ for odd i, and $|H_i| \geq 0$ for even i, then **H** is negative semi-definite. Here, H_i is the ixi matrix which is the i upper left-hand rows and columns of **H**. The positive definite and positive semi-definite is easy to remember. For the negative, think of $(-1)^i$. Starting with -1, it alternates from negative to positive. *If f(**x**) is additively separate, then you have a diagonal matrix and everything is much easier.*

Section 11.5: A **bordered Hessian matrix** is represented by $\bar{\mathbf{H}}$ is **H** with a row and column added to the upper and left sides. They are [0 f_1' f_2' ... f_n']. It can be useful because if $\bar{\mathbf{H}}$ is such that all $|\bar{H}_i| < 0$ for odd i and >0 for even i, then f is quasi-concave. (See Section 2.4 if you forgot that.) If $|\bar{H}_i| < 0 \ \forall$ i then f is quasi-convex. Note that $\bar{H}_i$ is H_i with the border added, so $\bar{H}_1$ is 2x2 not 1x1. *Therefore,* $|\bar{H}_1|$ *must be negative*. Therefore, for quasi-convexity you just remember that it is the negative the whole way through. **Homogeneous of degree k** is found by replacing all x_i with cx_i. If you can then get the c out of the new function resulting in the following f(c**x**) = c^kf(**x**) then it is homogenous of degree k. *The value of k is helpful. If k<1 then there is decreasing returns to scale **(DRTS)**, if k=1 then there are constant returns to scale **(CRTS)**, and if k>1 then there are increasing returns to scale **(IRTS)***. Note that you cannot take a positive monotonic transformation before finding the degree because that would change the degree.

Non-graded Assignment #10A to be reviewed with Assignment #10.

1) (10 points) Find the bordered Hessian for $f(x, y) = 4x^{1/2}y^{1/2.}$

2) (20 points) Find $\bar{\mathbf{H}}$ for f(x, y) = 5xy. Determine if f is quasi-concave, quasi-convex, or neither.

3) (20 points) Find the degree of homogeneity for the general Cobb-Douglas production function $Q = A*K^aL^b$. What does that tell you about a simple way to tell the returns to scale for a Cobb-Douglas? Now do the positive monotonic transformation where $\bar{Q} = (1/A^c)Q^c$ where c = 1/(a+b). Find the degree of homogeneity for that new function.

4) (10 points) Given that taking a positive monotonic transformation of a function does not affect the level curves, and given what we already learned about Cobb-Douglas functions in class, Question #2 and Question #3, what can you say about the quasi-concavity or quasi-convexity of all Cobb-Douglas functions? Explain your logic.

5) (20 points) Find $\bar{\mathbf{H}}$ for $f(x, y) = x^2 + y^2$. Determine if f is quasi-concave, quasi-convex, or neither.

6) (20 points) Find $\bar{\mathbf{H}}$ for $f(x, y) = (x + y)^2$. Determine if f is quasi-concave, quasi-convex, or neither.