Write your name on the cover of the test booklet and nowhere else. Failure to follow these directions will cost you 1 point. The test has 100 points (to be scaled up to 250 points) and is scheduled to take 50 minutes. Therefore, expect to spend 1 minute for every 2 points. For example, a 12-point question should take 6 minutes. I will allow some extra time, but I will not allow much.

Show all work on all questions.

1) (8 points) Do EITHER Part A OR Part B.

A) Suppose your utility function is given by $U(C, P) = 5C^{1/4}P^{1/6}$. What transformation could you do to make it easier to work with? Prove it is a legitimate transformation

B) Suppose your income this year was \$500, next three years was \$600, \$700, and \$800 respectively. The interest rate is 5%. Your utility function is given by $U = C_0^{1/5}C_1^{1/5}C_2^{1/5}C_3^{1/5}C_4^{1/5}$. Set up the Lagrangian and state how you got it. **Do not solve it.**

2) (12 points) Do EITHER Part A OR Part B.

A) Is the $U(X, Y) = 8X^{1/4}Y^{1/4}$ a legitimate utility function? Prove your answer is true using the formal tests.

B) What are **two** of the tests for a legitimate utility function? Explain the economics as to why we have that test.

3) (20 points) Find all Nash equilibria in the following matrix, if any exist. Prove that you found all and prove they are Nash equilibria. Does either firm have a dominant strategy? How can you tell? Find the cooperative equilibrium. Explain how you found it. What are the two players' secure strategies? How did you find them?

Payoff Matrix		Astros	
		High price	Medium Price
Yankees	High Price	6 2	7 8
	Low Price	9 5	3 4

(20 points each) Answer **THREE** of the following.

4) Suppose the production function is given by $Q(K,L) = 4K^{1/2}L^{1/2}$. If the wage rate is \$4/L and the rental rate is \$16/K, then find the total cost function.

5) Suppose the Cournot style industry demand curve is given by $P = 102 - 3Q_I$. Firm i's cost function is given by $TC_i = Q_i^2 + 3Q_i + 7$. Find the two firms' outputs and the market price.

6) Suppose the Von Stackelberg follower's best-response-function is given by $Q_F = 22 - \frac{1}{2}Q_L$. The leader's total cost function is given by $TC_L = 3Q_L^2 + Q_L + 10$. If the industry demand curve is given by $P = 205 - 2Q_I$, then what are the two firms' outputs and price?

7) Suppose your utility function is given by $U(B, F) = 6B^{1/3}F^{1/2}$. The price of a baseball is \$4 per ball and the price of a football is \$6 per ball. Your income is \$640. How much of each would you buy? Approximately, what is your marginal utility of income?

8) Suppose your utility is given by $U(B, E, M) = 4B^{1/4}E^{1/4}M^{1/4}$. Business and Economics classes take 3 hours while Mathematics classes take 4 hours. All courses cost \$1000 each. You have 16 hours and \$4000 to spend. Set up the Lagrangian for maximizing utility. Take the derivatives, but **do not solve** the system.