

Write your name on the cover of the test booklet and nowhere else. Failure to follow these directions will cost you 1 point. The test has 100 points (to be scaled up to 250 points) and is scheduled to take 50 minutes. Therefore, expect to spend 1 minute for every 2 points. For example, a 12-point question should take 6 minutes. I will allow some extra time, but I will not allow much.

Show all work on all questions.

1) (8 points) Do EITHER Part A OR Part B.

A) Suppose your income this year was \$1500, next three years was \$1700, \$1900, and \$2100 respectively. The interest rate is 10%. Your utility function is given by $U = C_0^{1/5} C_1^{1/5} C_2^{1/5} C_3^{1/5} C_4^{1/5}$. Set up the Lagrangian and state how you got it. **Do not solve it.**

B) Suppose your utility function is given by $U(C, P) = 5C^{1/3}P^{1/4}$. What transformation could you do to make it easier to work with? Prove it is a legitimate transformation

2) (12 points) Do EITHER Part A OR Part B.

A) What are **two** of the tests for a legitimate utility function? Explain the economics as to why we have that test.

B) Is the $U(X, Y) = 12X^{1/3}Y^{1/2}$ a legitimate utility function? Prove your answer is true using the formal tests.

3) (16 points) Answer EITHER Part A OR Part B.

A) Suppose your utility from learning (i.e. better grade) and video games is given by $U(L, V) = 8L^{1/4}V^{1/2}$. They both take the same amount of time, 1/hr per unit of studying and 1/hr per unit of playing video games. You have 15 hours of time awake when you are not eating. How much time do you spend on each activity? About how much additional utility would you get if you could cut your sleep by 2 hours?

B) Suppose your utility from hats and vests is given by $U(H, V) = 8H^{1/2}V^{1/4}$. Hats cost \$20/hat and vests cost \$80/vest. How many of each would you buy if you had \$480 to spend on hats and vests? (I realize that spending that much on hats and vests is unrealistic.) What is your marginal utility of income?

4) (20 points) Find all Nash equilibria in the following matrix, if any exist. Prove that you found all and prove they are Nash equilibria. Does either team have a dominant strategy? How can you tell? Find the cooperative equilibrium. Explain how you found it. What are the two players' secure strategies? How did you find them?

Payoff Matrix		Astros		
		High price	Medium Price	Low Price
Red Sox	High Price	10 4	9 2	8 6
	Low Price	7 1	12 5	11 3

5) (18 points) Answer EITHER Part A OR Part B.

A) Suppose the Cournot model has two firms. The industry demand curve is given by $P = 100 - 3Q_i$. The i^{th} firm's cost curve is given by $TC_i = 4 + 9Q_i + 2Q_i^2$. Find the firms' best response functions, the

equilibrium outputs, and price.

B) Suppose the Von Stackelberg follower's best response function is $Q_F = 10 - Q_L/2$. The industry demand is given by $P = 85 - 2Q_I$. The leader's cost function is given by $TC_L = 2 + Q_L + Q_L^2$. Find both firms' outputs and the price.

6) (26 points) Answer EITHER Part A OR Part B.

A) Suppose your utility function for apples, bananas, and pears is given by $U(A,B,P) = 8A^{1/2}B^{1/4}P^{1/8}$. The price of an apple is \$4/apple. The price of a banana is \$2/banana. The price of a pear is \$4/pear. It takes 2 minutes to skin an apple and 1 minute to peel a banana. However, pears are not peeled. If you have \$50 and can spend 15 minutes peeling fruit, then how many of each would you buy?

B) Suppose the production function is given by $Q = 6K^{1/3}L^{1/6}$. The wage rate is \$20/hour and capital rents for \$40/hour. Find the λ , total cost, marginal cost, and average total cost functions as a function of Q .