

Chapter 13.1 of the textbook for ECON 205 can also help you.

The Lagrangian method of solving constrained optimization is the most frequently used method in economics. If you have a function $f(x, y, z, \dots)$ and you want to either maximize or minimize it with respect to a constraint given by $g(x, y, z, \dots) = c$, then you setup the Lagrangian as:

$\mathcal{L} = f(x, y, z, \dots) + \lambda(c - g(x, y, z, \dots))$. Note that since $g(x, y, z, \dots) = c$, you have just added $\lambda * 0 = 0$. Adding zero does not change the value of the equation.

You now have one equation with one additional unknown, λ . Anytime you maximize or minimize, you take the partial derivative with respect to each variable, one at a time. Set each of the partial derivatives equal to zero. Do not forget to take the derivative with respect to λ . Then solve the equations simultaneously.

For example, if you want to minimize xy subject to $x + 3y = 6$, the Lagrangian equation is given by: $\mathcal{L} = xy + \lambda(6 - (x + 3y))$. I would distribute the negative sign through the parentheses, and you may want to distribute the λ too. I generally do not. This distribution gives: $\mathcal{L} = xy + \lambda(6 - x - 3y)$. Now take all three partial derivatives. (They are just like regular derivatives, but you treat the other variables as constants.) These can be written as either $\partial \mathcal{L} / \partial x$ or as \mathcal{L}'_x where x is the variable which you are taking the derivative with respect to. The results of the derivatives are given as: $\mathcal{L}'_x = y + \lambda(-1) = 0$, $\mathcal{L}'_y = x + \lambda(-3) = 0$, $\mathcal{L}'_\lambda = 6 - x - 3y = 0$. Note that when you take the derivative with respect to λ , you will get your original constraint again. If you rewrite it, you will get $6 = x + 3y$.

Given the way the equations usually work, it is almost always easiest to solve the first two equations for λ and then set them equal. Then solve for x or y and substitute into the last equation. Here you will get $y = \lambda$ and $x = 3\lambda \Rightarrow \lambda = x/3$. So $x/3 = y$ or $y = 3x$. Substitute that equation into the last equation and get: $6 - 3y - 3y = 0$. So $6 - 6y = 0$. Thus, $y = 1$, $x = 3$, and $\lambda = 1$. **I would double check your work** by substituting the values of x and y into your constraint to make sure that your answer is feasible, and substitute them back into both equations for λ .

λ represents the change in the value of the objective function (f) that results from a unit relaxation of the constraint (c). In this example, $f(x, y) = 3 * 1 = 3$. If the constraint is changed to $x + 3y = 7$, then λ tells us that the function's value will go up by approximately 1. (Reworking the example will show that f will actually increase by 1.083333.)