

Write your name on the cover of the test booklet and nowhere else. Enclose this sheet with the booklet. Failure to follow these directions will cost you 1 point. The test has 100 points (to be scaled up to 170 points) and is scheduled to take 50 minutes. Therefore, expect to spend 1 minute for every 2 points. For example, a 12-point question should take 6 minutes. I can give extra time but I will not give much.

**Show all work on all questions.**

1) (8 points) Answer EITHER Part A OR Part B.

- A) How is the test for concavity/convexity of a function related to the test for determining whether a stationary point is a local maximum or a local minimum? Explain why they have this relationship.  
 B) Explain the difference between a local maximum and a global maximum.

2) (8 points) Answer EITHER Part A OR Part B.

- A) A semi-common utility function is  $U(H) = \ln(H)$  where  $H$  is the number of hats. What is the marginal utility function? Briefly explain why you did what you did.  
 B) A function for the GDP at time  $t$  would be something like  $Y(t) = 1000e^{-0.2t}$ . How much does GDP change at time  $t=50$ ? You can leave  $e$  in your answer.

3) (10 points each) Find the first derivative of THREE of the following equations:

- A)  $F(Y) = (2Y+4)^5$  where  $Y = 6X^3$ . Find  $dF/dX$   
 B)  $F(X) = (X^2+5)/(4X+3)$   
 C)  $F(X) = 5X^3 + 4/X + \sqrt{X}$   
 D)  $F(X) = 4X^4 - 7X^3 + 6X^2 + 8 + 3X^{-2}$

4) (14 points) Answer EITHER Part A OR Part B.

- A) If a function is given by  $Y = X^3 - 30X^2 + 300X + 10$ , then find all stationary points. For each point, is it a local maximum, a local minimum, or a stationary point? Explain your logic.  
 B) Suppose a function was  $f(X) = \frac{1}{3}X^3 + \frac{1}{2}X^2 - 30X + 15$ . Find all stationary points. For each point, is it a local maximum, a local minimum, or a stationary point? Explain your logic.

5) (18 points) Suppose a firm is facing a demand of  $P = 217 - 2Q$  and its total cost function is given by  $TC = 17Q + 18Q^2$ . Find the profit maximizing output. Prove that it is the profit maximizing not profit minimizing.

6) (22 points) Answer EITHER Part A OR Part B.

- A) If a firm's demand curve is given by  $P = 242 - 2Q$  and its cost function is given by  $TC = 8Q^2 + 2Q + 10$ . Find their profit function. Suppose the building is only big enough to produce 10. Find the profit maximizing output. What is the shadow price of the constrained building size? Approximately how much would profits increase if the size of the building was increased to allow them to produce one more item?  
 B) Suppose a firm's demand curve is given by  $Q = 200 - 2P$ . Their total cost function is given by  $TC = 2Q$ . Find their total revenue and total cost functions as a function of  $P$ . Find the profit-maximizing price. Suppose the monopoly is constrained to charge less than 20. What is the shadow price of the price ceiling? How much could the profits increase if they could raise their price by 1?