

Write your name on the cover of the test booklet and nowhere else. Enclose this sheet with the booklet. Failure to follow these directions will cost you 1 point. The test has 100 points (to be scaled up to 170 points) and is scheduled to take 50 minutes. Therefore, expect to spend 1 minute for every 2 points. For example, a 12-point question should take 6 minutes. I can give extra time but I will not give much.

Show all work on all questions.

1) (4 points each) Suppose $F(X, Y, Z) = 3XYZ - 4X^2Y^2 + 5Z$. Find THREE of the following:

A) $\partial F / \partial X$

B) F_{XY}''

C) $F_{1,1}''$

D) $\frac{\partial^2 F}{\partial X \partial Z}$

2) (6 points) Answer EITHER Part A OR Part B.

A) We stated that if matrix B is matrix A with a multiple of one row added to another row, then $|A| = |B|$.

Prove this for $A = \begin{bmatrix} W & X \\ Y & Z \end{bmatrix}$ and $B = \begin{bmatrix} W+Y & X+Z \\ Y & Z \end{bmatrix}$.

B) We stated that if matrix B is matrix A with two rows swapped or two columns swapped, then $|A| =$

$-|B|$. Prove this for $A = \begin{bmatrix} W & X \\ Y & Z \end{bmatrix}$ and $B = \begin{bmatrix} X & W \\ Z & Y \end{bmatrix}$.

3) (10 points) Find the determinant of A when $A = \begin{bmatrix} 0 & 2 & 0 \\ -3 & 0 & 5 \\ 0 & -1 & 1 \end{bmatrix}$

4) (10 points) Suppose your utility function for clothing (C) and food (F) is given by $U(C, F) = 24C^{1/2}F^{1/3}$, then find the marginal utility of clothing (MUC). Find the slope of the MUC to determine if there is diminishing marginal utility of clothing.

5) (12 points) Set up the following system of equations in the $Ax = b$ format. Find A^{-1} and use that to solve the system. $4X + 3Y = 10$, $X - Y = -1$.

6) (14 points) Set up the following system of equations in the $Ax = b$ format. Use Cramer's Rule to solve the system. The first equation is the demand and the second is the supply. $Q = 10 - 0.2P$. $Q = P - 2$.

7) (16 points) Suppose $F(X, Y) = 3X^2Y + 4XY^3$. Find ∇F and $\nabla^2 F$.

8) (20 points) Answer EITHER Part A OR Part B.

A) Find both the adjoint and inverse of the matrix in Question #3, in other words $\text{adj}(A)$ and A^{-1} .

B) Suppose that producing \$1 worth of corn uses \$0.10 worth of corn and \$0.20 worth of energy. \$1 worth of energy uses \$0.3 units of energy. Set up the Leontief input-output matrix, A. Use that to find $I-A$. If you want to sell \$1260 worth of corn and \$630 worth of energy, then how much of each should you make? (You can use either $(I-A)^{-1}$ or Cramer's Rule.) If each \$1 of corn requires 3 units of labor and every \$1 worth of energy requires 1 unit of labor, then how much labor will you need?