

Write your name on the cover of the test booklet and nowhere else. Enclose this sheet with the booklet. Failure to follow these directions will cost you 1 point. The test has 240 points (to be scaled down to 200 points) and is scheduled to take 50 minutes. Therefore, expect to spend 1 minute for every 2 points. For example, a 12-point question should take 6 minutes. I can give extra time but I will not give much.

**Show all work on all questions.**

1) (8 points) Answer EITHER Part A OR Part B.

A) Without actually doing the calculations, find the determinant of  $\begin{vmatrix} 3 & 4 & 5 & 6 \\ 6 & 8 & 10 & 12 \\ 9 & 12 & 15 & 18 \\ 12 & 16 & 20 & 24 \end{vmatrix}$ . State how you knew the value.

B) What is the geometric interpretation of  $\partial F/\partial X$ ? Explain how  $\partial F/\partial X$  can be negative while  $\partial F/\partial Y$  is positive.

2) (10 points) What are the units, a.k.a. dimension, of EITHER price OR marginal cost? Explain your logic.

3) (12 points) Answer EITHER Part A OR Part B.

A) Find the distance between the points (-1, 4, -6, 8) and (2, -1, -5, 9). Show all work.

B) Plot the point (-3, 4, 2) Briefly explain how you did it.

4) (12 points) Find  $\text{trace}(A^T B)$  where  $A = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$ ,  $B = \begin{bmatrix} -1 & -2 \\ 4 & 0 \end{bmatrix}$

5) (12 points) Answer EITHER Part A OR Part B.

A) A function for the GDP at time  $t$  would be something like  $Y(t) = 1000e^{0.02t}$ . How much does GDP change at time  $t=50$ ? You can leave  $e$  in your answer.

B) A semi-common utility function is  $U(C) = \ln(C^2)$  where  $H$  is the number of cars. What is the marginal utility function? Briefly explain why you did what you did.

6) (14 points) Use the formal proof to determine the degree of homogeneity of the production function and whether it has increasing, decreasing, or constant returns to scale. Briefly state how you reached your conclusion.  $Q(L, K) = 6L^{1/3}K^{1/3}$ .

7) (16 points) Answer EITHER Part A OR Part B.

A) If  $TC = (Q^3)/3 - 15Q^2 + 250Q + 10$ , then find the minimum point of the marginal cost curve. Prove that it is a minimum not a maximum.

B) Find  $F'$  when  $F(X) = (X^2+3)^4/(3X^2+1)$

8) (16 points) Suppose your utility function is represented by  $U(C, F) = 12C^{1/2}F^{1/3}$ , where  $C$  represents clothing and  $F$  represents food. If you were to plot the indifference curves on a graph with clothing on the  $X$ -axis, then what is the slope of the indifference curve at (4,2)? What is the  $MRS_{CF}$ ? So how many units of clothing are you willing to give up for one unit of food, if you are at (4, 2)? Briefly state how you reached that conclusion.

9) (16 points) Answer EITHER Part OR Part B.

A) Use row operations to solve the system of equations.  $X + 3Y + Z = 10$ ,  $X + 2Y + 2Z = 10$ , and  $2X + 5Y + 3Z = 10$ .

B) Solve the system of equations using any method you choose. (These equations are respectively for the IS curve and the LM curve of the IS/LM diagram in *Intermediate Macroeconomics*.)  $r = 205 - .2Y$ , and  $r = -5 + .01Y$ .

10) (16 points) Find  $\mathbf{H}$  (the bordered Hessian) for  $f(x, y) = x^2 + y^2$ . Determine if  $f$  is quasi-concave, quasi-convex, or neither. Make sure you tell me the test for quasi-concave and the test for quasi-convex which use the bordered Hessian.

11) (16 points) Answer EITHER Part A OR Part B.

A) Suppose your utility function for clothing (C) and food (F) is given by  $U(C, F) = 24C^{1/2}F^{1/3}$ , then find the marginal utility of clothing (MUC). Find the slope of the MUC to determine if there is diminishing marginal utility of clothing.

B) Set up the following system of equations in the  $\mathbf{Ax} = \mathbf{b}$  format. Use Cramer's Rule to solve the system.  $Q = 2300 - 3P$ .  $Q = 22P - 200$ . They are demand and supply respectively.

12) (20 points) Answer EITHER Part A OR Part B.

A) Suppose that in a given year, 90% of the people of Pittsburgh stay in Pittsburgh and the remaining 10% move to New York City. 80% of New Yorkers stay in New York and the remaining 20% move to Pittsburgh. What is the P matrix? If initially there are 100 people in NYC and 50 in Pittsburgh, then how many people will be in each city in 2 years.

B) Suppose the population matrix  $P = \begin{bmatrix} .9 & .3 \\ .1 & .7 \end{bmatrix}$  and suppose the population of the two cities totals 100 people. What distribution of the people would result in the populations of the two cities being the same every year? In other words, what values of X and Y would result in  $\begin{bmatrix} .9 & .3 \\ .1 & .7 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} X \\ Y \end{bmatrix}$  and  $X+Y = 100$ . Hint: Do the multiplication out and then solve the system of equations.

13) (20 points) Answer EITHER Part A OR Part B.

A) Suppose \$1 worth of food (F) uses \$.10 worth of food and \$.10 worth of energy (E) to produce it. \$1 worth of energy uses \$.20 worth of energy to produce it. Create the Leontief Input-Output matrix. Find I-A matrix. If we want to sell \$720 worth of food and \$1440 worth of energy, then how much do we have to make? You can use any of the three methods of solving the equations (Cramer's Rule, (I-A)<sup>-1</sup>, and row operations on an augmented matrix).

B) Set up the equations  $2X + Y = 50$ ,  $X + Z = 30$ , and  $3X + 2Z = 70$  in the  $\mathbf{Ax}=\mathbf{b}$  format. Use the cofactor method to find  $A^{-1}$  and use that to solve the system of equations.

14) (24 points) Answer EITHER Part A OR Part B.

A) Draw a Venn diagram for Bethany College with four categories of students, tall (T), Short (S), male (M), and female (F). Given your diagram what percentage of Bethany students are in the following groups?  $T \cup F$ ,  $M \cap S$ , and  $\bar{T}$ . Briefly explain how you reached each conclusion.

B) Use the series formed by  $s_n = \sum_{t=1}^n \frac{FV}{(1+r)^t}$ . What is the  $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right|$ ? Is the series convergent or divergent? Explain your logic. **If convergent**, what does it converge to? Show all work. **If divergent**, what does that say about stock values?

15) (28 points) Answer EITHER Part A OR Part B.

A) Suppose a firm's demand curve is given by  $Q = 200 - 2P$ . Their total cost function is given by  $TC=2Q$ . Find their total revenue and total cost functions as a function of P. Find the profit-maximizing price. Suppose the monopoly is constrained to charge less than 20. What is the shadow price of the price ceiling? How much could the profits increase if they could raise their price by 1?

B) If a firm's demand curve is given by  $P = 144 - \frac{1}{2}Q$  and its cost function is given by  $TC = 2Q^2 + 4Q + 10$ . Find their profit function. Suppose the building is only big enough to produce 25. Find the profit maximizing output. What is the shadow price of the constrained building size? Approximately how much would profits increase if the size of the building was increased to allow them to produce one more item?