

Place your name on the back of this sheet of paper and nowhere else. Staple your answers face up on the front of this sheet of paper. Failure to follow these directions will cost you 1 point. If you use double-sided printing or print on the back of scrap paper, I will give you one additional point.

**Show all work for all questions.**

1) (10 points each) Find each of the derivatives asked for.

A)  $F(X) = 5X^3 + 4X + \ln(X)$ . Find  $F'(X)$  and  $F''(X)$ .

B)  $F(X) = 4X^{1/2} + 10X - 8$ . Find  $dF/dX$  and  $d^2F/dX^2$ .

C)  $F(X) = 10e^{3X} + 7X^2$ . Find  $F'(X)$  and  $F''(X)$ .

2) (15 points) Show all work for all parts. Suppose your utility function is given by  $U(Q) = 9Q^{2/3}$ . Find the  $MU(Q)$ . Find the slope of the  $MU(Q)$  function you just calculated. Is the slope of the  $MU$  function what you expected? Explain your logic. Suppose that your income is  $I$  and the price of  $Q$  is 20. What is the function  $Q(I)$ ? State how you found that. Using the chain rule, what is the marginal revenue of income?

3) (10 points) If the average total cost function is given as  $ATC \equiv TC(Q)/Q$ , then find the formula for the slope of the  $ATC$  function. Use that formula to prove that the  $MC$  curve must go through the minimum point of the  $ATC$  function.

4) (15 points) Suppose the cost of production is  $TC(L) = 10L + 5$ . Your production function is given by  $Q(L) = 3L^{2/3}$ . Find  $MC(L)$  and the change in output per worker, i.e.  $MP(L)$ . Given these two answers, what is the  $MC(Q)$  as a function of  $L$ ? (Remember to use the inverse rule.)

5) (10 points) Suppose your utility function is given by  $U(Q) = \ln(Q)$ . Find the  $MU(Q)$  and the slope of the marginal utility curve. Is the utility function strictly concave, concave, convex, strictly convex, or none of the above? Explain your logic.

6) (10 points) Suppose the population of a country is given by  $POP(t) = 300e^{0.01t}$ . What is the change in the population at time  $t$ ? Is the function strictly concave, concave, convex, strictly convex, or none of the above? Explain your logic.

7) (10 points) Suppose your demand function is defined as  $P(Q) = 30 - 2Q$ . Find the elasticity of demand for a quantity of 8. Hint: Write the formula for elasticity first and figure out what goes in where. Briefly explain what you did and why.