

Write your name on the cover of the test booklet and nowhere else. Enclose this sheet with the booklet. Failure to follow these directions will cost you 1 point. The test has 150 points (to be scaled up to 210 points) and is scheduled to take 75 minutes. Therefore, expect to spend 1 minute for every 2 points. For example, a 12-point question should take 6 minutes. I can give extra time but I will not give much.

**Show all work for all questions.**

1) (6 points each) find FOUR of the following derivatives.

- A)  $TR(Q) = 20Q - 2Q^2$ . What is MR?
- B)  $U(H) = 4H^{1/2}$ . Find the marginal utility of hats.
- C) What is the slope of the  $MU_H$  function from your answer to Part B?
- D)  $TR(Q) = P(Q) \cdot Q$ . If  $P(Q) = 40 - \frac{1}{2}Q$ , then use the product rule to find marginal revenue.
- E)  $TC = 30 - 10Q + Q^2$ . Find MC.
- F) Profits =  $-3Q^2 + 100Q - 9$ . Find the marginal profits function.
- G)  $Y = 5X^2 + 4X - 19$ . Find  $dY/dX$ .

2) (6 points each) For TWO of the following variables, find the dimensions, a.k.a. units. Briefly explain how you reached that conclusion.

- A) unemployment rate
- B) average total costs
- C) total revenue

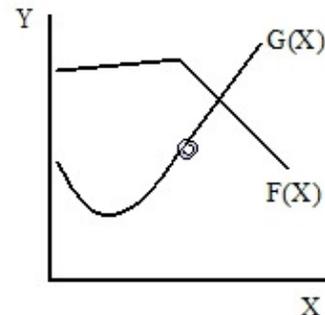
3) (6 points) Answer EITHER Part A OR Part B.

- A) If a curve is monotonically decreasing and bounded, must it have a limit? Explain your logic.
- B) When is  $e$  used when compounding? Why is that rarely used in economics.

4) (6 points) Find  $\lim_{n \rightarrow \infty} \left( \frac{n^3 + 5n + 6}{4n^3 + 2n^2} \right)$  or  $\lim_{n \rightarrow \infty} \left( \frac{3n^2 + 5n + 6}{n^3 + 2n^2} \right)$  Show work or briefly explain how you reached your conclusion.

5) (10 points) Answer EITHER Part A OR Part B.

- A) What is the relationship between marginal revenue and the graph of the total revenue function? Given that relationship, how can we find the maximum total revenue if we know the marginal revenue function? Explain your reasoning.
- B) In the graph to the right, are  $F(X)$  and  $G(X)$  differentiable over the whole domains of the respective functions? Explain your logic.



6) (10 points) Plot EITHER  $F(X) = \ln(X)$  over the domain  $[1, e]$  OR  $F(X) = 10X - X^2$  over  $[0, 10]$ .

7) (12 points) Answer EITHER Part A OR Part B.

- A) Plot  $(\infty, 4]$  and  $[5, 9)$ . Is either set compact? Explain your logic.
- B) Find the distance between  $(-2, 4, 5, -1)$  and  $(2, -6, 6, -3)$ .

8) Is on the back.

8) (12 points each) For TWO of the following sequences, find the first four terms. Does the sequence have a limit as  $n$  approaches  $\infty$ ? If yes, then find it. If no, then explain how you know there is no limit. Is it bounded or definitely divergent? Explain your logic.

A)  $a_n = 1/n$

B)  $a_n = n/(n+1)$

C)  $a_n = 4 \cdot (-1)^n$

9) (14 points) Answer EITHER Part A OR Part B.

A) Suppose that you are considering buying a bond which will pay you \$100 a quarter for 20 years. At the end, it will pay you \$10,000. If you want a 4% annual return, then what is the formula which determines the most you would pay for it? Explain how you determined what values to put into the equation for each variable. (Warning: this is quarterly payments not annual payments.)

B) Economists have come up with the idea of a bond that is infinitely lived, in other words, it never matures. Therefore, it pays the same interest every year forever. Suppose that such a bond pays \$1000 a year starting in one year. If you want a 10% return, what is the general formula for  $a_n$  (where  $a_n = PV$  of the one payment in  $n$  years) the sequence of the PVs? Prove the series  $s_1, s_2, s_3, \dots$  converges. If the current price of the bond is  $\lim_{n \rightarrow \infty} (s_n)$ , then how much will the bond be selling for? Show all work and briefly explain how you found the value.

10) (16 points) Answer EITHER Part A OR Part B.

A) Draw a graph of a function over the domain  $[0, 10]$ , which is concave, but not strictly concave. State how you know it is concave but not strictly concave. Is your function invertible? Explain your logic.

B) Suppose that a utility function for pencils (P) and balloons (B) is given by  $U=120/(PB)$ . (Actually, this is not a valid utility function.) Plot one indifference curve. State how you found it. Is this function strictly quasi-concave, quasi-concave, quasi-convex, strictly quasi-convex, or none of the above? Explain your logic.

11) (16 points) Answer EITHER Part A OR Part B.

A) Suppose that the universal set  $U = \mathbf{Z}_+$ ,  $A = \{2, 3, 4, 7, 10, 13\}$ , and  $B = \{1, 3, 5, 7, 9\}$ . Find the following sets  $A \cup B$ ,  $\bar{A} \cap B$ ,  $A - B$ , and  $\bar{A} \cap A$ . Briefly state how you got each answer.

B) Draw a Venn diagram where the universal set is Bethany College Students. Draw areas for members of fraternities or sororities (G for Greek), and athletes (A). Given your diagram, what percentage of the students are in the sets  $G \cap A$  and  $\bar{A} \cup B$ . State how you came up with those answers.