

This review sheet is intended to cover everything that could be on the exam; however, it is possible that I will have accidentally left something off. You are still responsible for everything in the chapters covered except anything that I explicitly say you are not responsible for. Therefore, if I left something off of this sheet, it can still be on the exam. There will be no multiple-choice questions. Most of the questions will be like the ones in the homework assignments, and possibly a few definition questions, but I am more likely to ask questions that make you use the definitions rather than recite them. I will probably ask one of the questions from the book at the end of the chapters.

When I write the final, I try to draw questions evenly from all parts of the course. However, Sections 11.4 - 11.5 had not be covered before, so there will be slightly more emphasis on those three sections.

The review session will probably be 5/11 or 5/12. The final is Tuesday 5/13 at 3:00

11.4 starting at Page 441: The easiest way to do the tests for strictly concave, concave, strictly convex, and convex, is to use the theorem on Page 443, if \mathbf{H} is **positive definite** then $f(\mathbf{x})$ is strictly convex, if \mathbf{H} is **negative definite** then $f(\mathbf{x})$ is strictly concave, if \mathbf{H} is **positive semi-definite** then $f(\mathbf{x})$ is convex, and if \mathbf{H} is **negative semi-definite** then $f(\mathbf{x})$ is concave. Unfortunately, this requires a little bit of Section 10.3 which we skipped. Basically if $|H_i| > 0$ for all i , \mathbf{H} is positive definite. If $|H_i| \geq 0$ for all i , \mathbf{H} is positive semi-definite. If $|H_i| < 0$ for odd i , and $|H_i| > 0$ for even i , then \mathbf{H} is negative definite. If $|H_i| \leq 0$ for odd i , and $|H_i| \geq 0$ for even i , then \mathbf{H} is negative semi-definite. Here, H_i is the i th upper left-hand row and column of \mathbf{H} . The positive definite and positive semi-definite is easy to remember. For the negative, think of $(-1)^i$. Starting with -1 , it alternates from negative to positive. If $f(\mathbf{x})$ is additively separate, then you have a diagonal matrix and everything is much easier.

Section 11.5: A **bordered Hessian matrix** is represented by $\bar{\mathbf{H}}$ is \mathbf{H} with a row and column added to the upper and left sides. They are $[0 \ f_1' \ f_2' \ \dots \ f_n']$. It can be useful because if $\bar{\mathbf{H}}$ is such that all $|\bar{H}_i| < 0$ for odd i and > 0 for even i , then f is quasi-concave. (See Section 2.4 if you forgot that.) If $|\bar{H}_i| < 0 \ \forall i$ then f is quasi-convex. Note that \bar{H}_i is H_i with the border added, so \bar{H}_1 is 2×2 not 1×1 . Therefore, $|\bar{H}_1|$ must be negative. Therefore, for quasi-convexity you just remember that it is the negative the whole way through. **Homogeneous of degree k** is found by replacing all x_i with cx_i . If you can then get the c out of the new function resulting in the following $f(\mathbf{c}\mathbf{x}) = c^k f(\mathbf{x})$ then it is homogeneous of degree k . *The value of k is helpful. If $k < 1$ then there is decreasing returns to scale (DRTS), if $k = 1$ then there are constant returns to scale (CRTS), and if $k > 1$ then there are increasing returns to scale (IRTS).* Note that you cannot take a positive monotonic transformation before finding the degree because that would change the degree.

Non-graded Assignment #10B to be reviewed during the last class.

- 1) (10 points) Find the bordered Hessian for $f(x, y) = 4x^{1/2}y^{1/2}$.
- 2) (20 points) Find $\bar{\mathbf{H}}$ for $f(x, y) = 5xy$. Determine if f is quasi-concave, quasi-convex, or neither.
- 3) (20 points) Find the degree of homogeneity for the general Cobb-Douglas production function $Q = A^*K^aL^b$. What does that tell you about a simple way to tell the returns to scale for a Cobb-Douglas? Now do the positive monotonic transformation where $Q = (1/A^c)Q^c$ where $c = 1/(a+b)$. Find the degree of homogeneity for that new function.
- 4) (10 points) Given that taking a positive monotonic transformation of a function does not affect the level curves, and given what we already learned about Cobb-Douglas functions in class, Question #2 and Question #3, what can you say about the quasi-concavity or quasi-convexity of all Cobb-Douglas functions? Explain your logic.
- 5) (20 points) Find $\bar{\mathbf{H}}$ for $f(x, y) = x^2 + y^2$. Determine if f is quasi-concave, quasi-convex, or neither.
- 6) (20 points) Find $\bar{\mathbf{H}}$ for $f(x, y) = (x + y)^2$. Determine if f is quasi-concave, quasi-convex, or neither.