

Write your name on the cover of the test booklet and nowhere else. Enclose this sheet with the booklet. Failure to follow these directions will cost you 1 point. The test has 100 points (to be scaled up to 225 points) and is scheduled to take 50 minutes. Therefore, expect to spend 1 minute for every 2 points. For example, a 12-point question should take 6 minutes. I can give some extra time.

Show all work for all questions.

1) (4 points) Answer EITHER Part A OR Part B.

A) Suppose $Q = f(P)$. What is the slope of the demand curve? (Remember what is on the vertical axis.)

B) A perfectly competitive firm faces a demand curve of $P = 20$. Find the marginal revenue function.

2) (8 points each) Find the slope of TWO functions and determine if it is concave, convex, or neither. Briefly explain your logic.

A) $Y(X) = 4X^2 - 3X + 5$

B) $Y(X) = X^4 + 3X^2 + 9$

C) $Y(X) = X^3 - 6X^2 + 8$

3) (12 points) Answer EITHER Part A OR Part B.

A) Find all stationary points of $Y = X^3 - 6X^2 + 12X - 8$. For each one, determine if it is a local maximum, local minimum, or an inflection point.

B) Find all stationary points of $Y = X^4$. For each one, determine if it is a local maximum, local minimum, or an inflection point.

4) (16 points) For ONE of these companies, find the profit-maximizing quantity, price, and profits. Prove it is profit maximizing rather than minimizing or an inflection point.

A) $TC = 3Q^2 + 20Q + 7$, $P = 120 - 2Q$

B) $TC = \frac{1}{2}Q^2 + 30Q + 6$, $P = 50 - \frac{1}{2}Q$

5) (16 points) Answer EITHER Part A OR Part B.

A) The population at time t , $P(t) = 400 * e^{0.02t}$. Find the derivative with respect to time. Use that to find the percent growth rate of the population. Is this function concave, convex, or neither? Explain your logic.

B) Sometimes economists say the utility of pants (P) is given by $U(P) = \ln(6P)$. Find the marginal utility function, the slope of the marginal utility function, and determine if the marginal utility function is concave, convex, or neither. Explain your logic.

6) (16 points) Answer EITHER Part A OR Part B.

A) Suppose a demand curve is given by $P(Q)$. What are the total revenue function, marginal revenue function, and average revenue function? Briefly explain what you did.

B) Suppose the total product of labor (TPL) is given by $TPL(L)$. Find the average product of labor (APL) function, marginal product of labor (MPL) function, and the slope of the APL. Briefly explain what you did.

7) (20 points) Answer EITHER Part A OR Part B.

A) Suppose a monopoly has an inverse demand curve of $Q = 98 - \frac{1}{2}P$ and a cost function of $TC = 4Q$. They are restricted to charging less than \$80/unit. Find their profit-maximizing price. What is the shadow price of the constraint. If the price was allowed rise \$2/unit, approximately how much would the profits rise.

B) Suppose a firm has an inverse demand of $Q = 1120 - 2P$ and a total cost function of $TC = 2Q^2 + 10Q + 9$. They are constrained to produce no more than 100 items. Find the constrained profit-maximizing output. What is the shadow price of the constraint? If the quota was increased by 3, approximately how much would the profits increase?