

Write your name on the cover of the test booklet and nowhere else. Enclose this sheet with the booklet. Failure to follow these directions will cost you 1 point. The test has 100 points (to be scaled up to 225 points) and is scheduled to take 50 minutes. Therefore, expect to spend 1 minute for every 2 points. For example, a 12-point question should take 6 minutes. I can give some extra time.

**Show all work for all questions.**

1) (10 points each) Use the following matrices to do FOUR of the following operations.

$$A = \begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix}, B = \begin{bmatrix} -2 & -5 \\ 0 & 10 \end{bmatrix}, C = \begin{bmatrix} -3 & 6 \\ -1 & -4 \end{bmatrix}, D = \begin{bmatrix} 20 \\ -10 \end{bmatrix}, E = \begin{bmatrix} 7 & 8 \end{bmatrix}$$

- A) Trace(DE)
- B)  $2A - C$
- C)  $2B + C^T$
- D) Trace(A+B)
- E)  $D^T E^T$  and ED
- F)  $D^T B$

2) (10 points) Answer EITHER Part A OR Part B.

A) Find the value for X which makes the matrix below idempotent. (Hint: use the definition of idempotent to find equations involving X. Solve any of those equations to get the answer.)

$$\begin{bmatrix} 3 & 1 \\ -6 & X \end{bmatrix}$$

B) For the system of equations  $X + Y = 0$ , and  $3X - 3Y = 0$ , determine if there are more solutions than just the trivial solution. If there are more solutions, find the formula for them. If there are not, then prove the trivial solution is the only solution.

3) (12 points) Answer EITHER Part A OR Part B.

A) Estimate the solution to the following system of equations by graphing.  $Q_D = 20 - 2P$ ,  $Q_S = 10P - 20$ .

B) Use elimination method to solve the following system of equations. SRAS:  $Y = 100 + 15P$ , AD:  $Y = 300 - 5P$ .

4) (14 points) Answer EITHER Part A OR Part B.

A) Suppose that 70% of people who start in Bethany live in Bethany the next year and the rest move to Pittsburgh. 90% of people who start in Pittsburgh stay there the next year and the rest move to Bethany. Setup the population migration matrix P. If the initial populations are 100 for Bethany and 1000 for Pittsburgh, then set up the population vector and multiply them to find out how many people are in each location next year.

B) Suppose that \$1 worth of food uses \$.30 worth of food and \$.10 worth of capital. \$1 worth of capital uses \$.20 of each. Set up the Leontief Input-Output Matrix. Set up a vector to show that you want to produce 1000 units of food and 200 units of capital. Multiply the vector and the matrix to find out how much of each will be used up.

5) (16 points) Answer EITHER Part A OR Part B.

A) Solve following system of equations any way you want.  $X + Y + Z = 10$ ,  $3X + 3Y + Z = 20$ ,  $2X + 2Y = 10$ .

B) Solve following system of equations any way you want.  $X + 2Y + 3Z = 10$ ,  $2X - Y + Z = 0$ ,  $5X + 5Z = 12$ .

6) (18 points) For EITHER the equations in Part A OR the equations in Part B, set them up as a partitioned matrix. Use row operations to solve the system. Tell me both the prices and quantities.

A)  $Q_{D2} = 40 - 2P_2 - P_3$ ,  $Q_{S2} = -5 + P_2$ ,  $Q_{D3} = 37.5 + P_1 - 3P_2 - \frac{1}{2}P_3$ ,  $Q_{S3} = -2.5 + \frac{1}{2}P_3$ ,  $Q_{D1} = 29.5 - \frac{1}{2}P_1 - P_2 - P_3$ ,  $Q_{S1} = -0.5 + \frac{1}{2}P_1$

B)  $Q_{D2} = 45 - P_2 - 2P_3$ ,  $Q_{S2} = -5 + P_2$ ,  $Q_{D3} = 27.5 + P_1 - 2P_2 - \frac{1}{2}P_3$ ,  $Q_{S3} = -2.5 + \frac{1}{2}P_3$ ,  $Q_{D1} = 39.5 - \frac{1}{2}P_1 - 2P_2 - P_3$ ,  $Q_{S1} = -0.5 + \frac{1}{2}P_1$