

Write your name on the cover of the test booklet and nowhere else. Enclose this sheet with the booklet. Failure to follow these directions will cost you 1 point. The test has 100 points (to be scaled up to 170 points) and is scheduled to take 50 minutes. Therefore, expect to spend 1 minute for every 2 points. For example, a 12-point question should take 6 minutes. I can give extra time but not much.

**Show all work on all questions.**

1) (6 points) Find  $dY/dX$  for EITHER  $Y(X) = (X^2 + 1)e^X$  OR  $Y(X) = (2X^2+1)/(3X-3)$

2) (6 points) Answer EITHER Part A OR Part B.

A) Suppose total utility is given by  $U(Q)$ . What two pieces of information does  $dU/dQ$  tell us? What two pieces of information does  $d^2U/dQ^2$  tell us? You can just state the information.

B) If  $f(X) = \ln(X^{1/2})$ , then what are  $f'$  and  $f''$ ?

3) (10 points each) Find the slope of TWO functions and determine if it is concave, convex, or neither.

Briefly explain your logic.

A)  $Y(X) = 4X^2 - 3X + 5$

B)  $Y(X) = 2X^3 + 6X^2 + 9X$

C)  $Y(X) = X^4 - 6X^2 + 8$

4) (12 points) Answer EITHER Part A OR Part B.

A) Find all stationary points of  $Y = 2X^3 - 15X^2 + 36X - 8$ . For each one, determine if it is a local maximum, local minimum, or an inflection point.

B) Find all stationary points of  $Y = X^5$ . For each one, determine if it is a local maximum, local minimum, or an inflection point.

5) (12 points) Answer EITHER Part A OR Part B.

A) Draw a graph which is continuous, but not differentiable. Explain how you know it is not differentiable.

B) Draw a graph which has a local maximum which is not a global maximum. Explain how you know it fits this definition.

6) (20 points) Answer EITHER Part A OR Part B.

A) Suppose a demand curve is given by  $Q(P) = a - bP$ . What are the total revenue function, marginal revenue function, and average revenue function? Briefly explain what you did. Prove the MR line has twice the slope of the demand curve.

B) Suppose the total product of labor (TPL) is given by  $TPL(L)$ . Find the average product of labor (APL) function, marginal product of labor (MPL) function, and the slope of the APL. Briefly explain what you did.

7) (24 points) Answer EITHER Part A OR Part B.

A) Suppose a monopoly has a demand curve of  $Q = 31 - \frac{1}{4}P$  and a cost function of  $TC = 12Q$ . They are restricted to charging less than \$60/unit. Find their profit maximizing price. What is the shadow price of the constraint. If the price was allowed rise \$2/unit, approximately how much would the profits rise.

B) Suppose a firm has an inverse demand of  $P = 300 - \frac{1}{4}Q$  and a total cost function of  $TC = \frac{1}{4}Q^2 + 50Q + 10$ . They are constrained to produce no more than 200 items. Find the constrained profit maximizing output. What is the shadow price of the constraint? If the quota was increased by 10, approximately how much would the profits increase?