

Write your name on the cover of the test booklet and nowhere else. Enclose this sheet with the booklet. Failure to follow these directions will cost you 1 point. The test has 100 points (to be scaled up to 170 points) and is scheduled to take 50 minutes. Therefore, expect to spend 1 minute for every 2 points. For example, a 12-point question should take 6 minutes. I can give extra time but not much.

**Show all work on all questions.**

1) (8 points) Do EITHER the equation in Part A OR the equation in Part B, find  $\frac{\partial^2 Q}{\partial K \partial L}$

A)  $Q(K, L) = 16K^{1/2}L^{1/4}$

B)  $Q(K, L) = \ln(4KL)$

2) (12 points) For EITHER the utility function in Part A OR the function in Part B, find a transformation which would make the function easier to deal with. Prove that it is a valid transformation. Find the transformation of the utility function.

A)  $U(H, J) = 30H^{2/3}J^{1/4}$

B)  $U(K, L, M) = 8K^{1/2}L^{1/4}M^{1/4}$

3) (14 points) Suppose \$1 worth of energy uses 10¢ of energy and \$1 of food uses 20¢ of energy and 30¢ of food. Set up the Leontief Input-Output Matrix,  $\mathbf{A}$ . Find  $(\mathbf{I}-\mathbf{A})^{-1}$  and use that to calculate how much energy and food needs to be produced if we want to sell \$88 of energy and \$70 of food.

4) (14 points) Answer EITHER Part A OR Part B.

A) Suppose your utility function for hats and coats is given by  $U(C, H) = 20C^{3/5}H^{2/5}$ . Use the implicit function theorem to find the slope of the indifference curve through (8, 3)

B) Suppose we had a general form of a utility function for bananas and soda,  $U(B, S)$ . Find the slope of the indifference curve using the implicit function theorem. The corresponding budget constraint is given by  $B \cdot P_B + S \cdot P_S = 100$ . Find its slope using the implicit function theorem.

5) (16 points) For EITHER of the following utility functions, find  $\nabla U$  and  $\mathbf{H}U$  and then determine if it is strictly convex, convex, concave, strictly concave, or none of the above. Briefly explain how you reached your conclusion.

A)  $U(R, T, V) = \ln(R) + \ln(T) + \ln(V)$

B)  $U(M, N) = 4M^{3/2}N^2$

6) (16 points) Put the following information into the  $\mathbf{Ax}=\mathbf{b}$ . Use Cramer's rule to solve the system. (These are supply and demand for a good.)  $5P + Q = 150$ .  $20P - Q = 100$ . 1 bonus point if you tell me which is the demand curve and which is the supply curve and prove your assertion.

7) (20 points) Find 
$$\begin{vmatrix} 7 & 2 & 8 \\ 3 & 0 & 6 \\ 5 & 1 & 0 \end{vmatrix}$$