Wilfrid W. Csaplar Jr. Quantitative Methods... Review Sheet for Exam 2

This review sheet is intended to cover everything that could be on the exam; however, it is possible that I will have accidentally left something off. You are still responsible for everything in the chapters covered except anything that I explicitly say you are not responsible for. Therefore, if I left something off of this sheet, it can still be on the exam. There will be no multiple-choice questions. Most of the questions will be like the ones in the homework assignments, and possibly a few definition questions, but I am more likely to ask questions that make you use the definitions rather than recite them. I will probably ask one of the questions from the book at the end of the chapters.

The review session's time will be determined, probably Sunday 3/26 in the normal room. Even though we will have covered Chapters 7.1 and possibly part of 7.2, they will not be on the test.

Chapter 5.1 - 5.2: Know what **marginal analysis** means. What is a **tangent line** and how does that relate to the slope of the line? Note that the limit of the slope of the **secant line** as Δx approaches zero is the slope of the tangent line. Know the definitions of **derivative** and **total differential**. Note that marginal cost is the derivative of total cost function.

5.3: When is a function **differentiable** over [a, b]? *If both the function and the plot of the slope are continuous over (a, b).*

Chapter 5.4: **Rules for differentiation**. Remember these rules f(X) = c, then f'(X) = 0. If f(X) = mX then f'(X)=m. If f(X) = X^n then f'(X) = nX^{n-1}. If g(X) = c*f(X) then g'(X) = c*f'(X). If h(X) = g(X) + f(X) then h'(X) = g'(X) +f'(X) and applies to adding even more terms. **Product rule** if h(X) = g(X)*f(X) then h'(X) = f'(X)*g(X) + g'(X)*f(X) which can also be used with division by defining k(X) = 1/f(X) or you can use the **division rule**, a.k.a., **quotient rule**. If h(X) = f(X)/g(X) then h'(X) = [f'(X)*g(X) – g'(X)*f(X)]/[g(X)]2. If you have y = f(U) and U = g(X) then h(X) = f(g(X)) and using the **chain rule** h'(X)=f'(U)*g'(X). If the inverse of Y = f(X) is X = g(Y), then using the **inverse rule**, g'(Y)=1/f'(X). If f(X) = e^x then f'(X) = e^x. If f(X) = ln(X) then f'(X) = 1/X.

Chapter 5.5: Know the symbols for **second derivative** or third etc. A function is **convex** if f''(X) $\geq$ 0 and it is **strictly convex** if f''(X) >0 at all points or all but one point. Reverse the inequalities for **concave** and **strictly concave**. Note that the second derivative test tells us if we found a maximum or a minimum.

Section 6.1: What are meant by **extreme values, unconstrained, constrained, global maximum, local maximum, global minimum, local** and **minimum**? The difference between global and local maximum and minimum are global is for all x, while local is for $\hat{x} - \varepsilon \leq x \leq \hat{x} + \varepsilon$. **First order condition** is f'(x*) =0 is a necessary condition for a local maximum, local minimum, and an **inflection point** (because it gives a **stationary point**) but not a sufficient condition. Note that most of this section is just analyzing different examples. Understand what f'(x) $\neq$ 0, $\forall$ x means. (No maximum and no minimum.)

Section 6.2: **Second-order conditions** can determine if it is a maximum or minimum. $f''(x^*)<0$ is maximum and $f''(x^*) > 0$ is a minimum. $f''(x^*) = 0$ could be either of those or an inflection point. Do not worry about the Taylor Series. Like Section 6.1, much of this chapter is just examples.

Section 6.3: Optimization **over an interval** is just like the local optimization except you have two differences. *First, check to make sure any extreme points you find are actually in the interval.* For example, finding the minimum of $f(x) = x^2 + 2x$, for $x \in [0, 100]$ will give a minimum at $x = -1$, which is not in the interval. *Second, check the limits of the interval.* In other words, check f(0) and f(100) in this example. If the maximum or minimum is inside the interval, then it is an **interior solution** and if it is not, then it is a **corner solution**. If there is a **constraint** that says $x \leq L$ or $x \geq L$, then when L is a **binding constraint**, you get a corner solution. The **shadow price of the constraint L** is the value of $f'(L)$.

Non-graded Assignment #5A to be reviewed with Assignment #5.

Show all work on all questions.

1) (25 points) Suppose a firm has an inverse demand of $P = 300 - \frac{1}{4}Q$ and a total cost function of $TC = \frac{1}{4}Q^2 + 50Q + 10$. They are constrained to produce no more than 200 items. Find the constrained profit maximizing output. What is the shadow price of the constraint? If the quota was increased by 10, approximately how much would the profits increase?

2) (25 points) Suppose a monopoly has a demand curve of $Q = 31 - \frac{1}{4}P$ and a cost function of $TC = 12Q$. They are restricted to charging less than \$60/unit. Find their profit maximizing price. What is the shadow price of the constraint. If the price was allowed rise \$2/unit, approximately how much would the profits rise.

3) (15 points) Find all stationary points for $F(X) = (\frac{1}{4})X^4 + (\frac{1}{3})X^3 - 8.5X^2 - 15X + 30$. Now I want you to determine if each one is a local maxima, local minima, or a inflection point.

4) (20 points) Suppose a firm has a total cost curve of $TC = Q^3 - 60Q^2 + 900Q$. They want to minimize the ATC but were constrained to produce no more than 25. Find the point which minimizes the ATC. What is the shadow price of the constraint?

5) (15 points) Maximize $Y = 5X^2 - 100X + 20$ over the interval [0, 30].