Wilf Csaplar Jr. Economics 205 Homework #8 **Due Wednesday 4/8 by 11:00 AM**

The sentences I am typing is so that you can follow what I am doing. You did not need to type them unless I ask you to explain.

1) (15 points) Write the following system of equations in the **Ax** = **b** form. Find $\mathbf{A}^{-1}$ and use that to solve the equations. $Q_D = 2000 - 50P$ $Q_S = 100P - 1000$.
We need them in the form where the Q and P are on the left and the constant is on the right. So, I add 50P to both sides of the first equation and subtract 100P from both sides of the second equation. Noting that $Q_D = Q_S$ means we get 50P +Q = 2000 and -100P +Q = -1000. I like to have the variables be in alphabetical order, so that gives us.
$\begin{bmatrix} 50 & 1 \\ -100 & 1 \end{bmatrix}\begin{bmatrix} P \\ Q \end{bmatrix} = \begin{bmatrix} 2000 \\ -1000 \end{bmatrix}$ For a 2x2, we swap the main diagonal and change the signs on the off diagonal the divide by |**A**| to get $\mathbf{A}^{-1}$. |**A**| = 50*1-(-100)*1 = 50+100 = 150. So $\mathbf{A}^{-1}$ =
$\begin{bmatrix} 1 & -1 \\ 100 & 50 \end{bmatrix}/150$. So we multiply

$$\begin{bmatrix} 1 & -1 \\ 100 & 50 \end{bmatrix}\begin{bmatrix} 2000 \\ -1000 \end{bmatrix}/150 = \begin{bmatrix} 2000+1000 \\ 200{,}000-50{,}000 \end{bmatrix}/150 = \begin{bmatrix} 3000/150 \\ 150.000/150 \end{bmatrix} = \begin{bmatrix} 20 \\ 1000 \end{bmatrix}$$

Therefore, the price is 20 and the quantity is 1000.

2) (15 points) Write the following system of equations in the **Ax** = **b** form. Find $\mathbf{A}^{-1}$ and use that to solve the equations. r = 30 - Y/100 r = 5 + Y/400.
As in #1, we move the Y terms to the left and get r +Y/100 = 30 and r-Y/400 = 5. Since do not like multiplying fractions (which would happen if we did the determinant in this form), I will multiply both sides of the first equation by 100 and the second equation by 400. This gives us 100r +Y = 3000 and 400r - Y = 2000. Putting that into the **Ab**=**x** form gives us:
$\begin{bmatrix} 100 & 1 \\ 400 & -1 \end{bmatrix}\begin{bmatrix} r \\ Y \end{bmatrix} = \begin{bmatrix} 3000 \\ 2000 \end{bmatrix}$ Again $\mathbf{A}^{-1}$ is achieve by switching the diagonal elements and changing the sign on the off diagonal elements because this is 2x2. Then we must divide by the determinant. (Bigger matrices require cofactors etc.) The determinant is 100*(-1)-(400)*1 = -500. So our $\mathbf{A}^{-1}$ is $\begin{bmatrix} -1 & -1 \\ -400 & 100 \end{bmatrix}/(-500)$ This gives us $\begin{bmatrix} -1 & -1 \\ -400 & 100 \end{bmatrix}\begin{bmatrix} 3000 \\ 2000 \end{bmatrix}/(-500)$.
Since it does not matter when we divide by -500, I will divide the 3000 and 2000 by -500 and get -6 and -4. So
$\begin{bmatrix} r \\ Y \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ -400 & 100 \end{bmatrix}\begin{bmatrix} -6 \\ -4 \end{bmatrix} = \begin{bmatrix} 6+4 \\ 2400-400 \end{bmatrix} = \begin{bmatrix} 10 \\ 2000 \end{bmatrix}$ So r = 10 and Y = 2000. Double check your answers in the equation. The first time I did this question, I had two sign swaps and my test said something was wrong. The second time, I accidentally typed the -4 as -5. That screwed things up. This time testing gives 10 = 30 - 2000/100 ✔ and 10 = 5 +2000/4. ✔

3) (10 points) For a generic 2x2 matrix, prove that swapping the two rows will change the sign of the determinant.

This is asking us to prove |**A**| = -|**B**| when $A = \begin{bmatrix} w & x \\ y & z \end{bmatrix}, B = \begin{bmatrix} y & z \\ w & x \end{bmatrix}$. |**A**| = wz-xy similarly |**B**| = yx-wz. Note that the signs on both terms changed. So |**A**| = -|**B**|

4) (5 points) Find |**A**| for **A**= $\begin{bmatrix} 4 & 9 & 3 & -1 & 7 \\ 0 & 2 & 5 & 1 & 5 \\ 0 & 0 & -1 & 8 & 3 \\ 0 & 0 & 0 & 10 & 7 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ Hint: It is only 5 points. Because this is a triangular matrix, the determinant is the product of the elements on the diagonal. So it is 4*2*(-1)*10*1 = -80. (This is an upper triangular matrix because all of the numbers below the diagonal are zero. Lower triangular matrices also have a determinant of the product of the elements on the diagonal. A lower triangular matrix has all elements above the diagonal = 0.

6) (15 points) Suppose **A** = $\begin{bmatrix} 2 & -2 \\ 4 & 5 \end{bmatrix}$. Find |**A**|. Suppose **B** is gotten by subtracting twice the first row from the second row. Find |**B**|.

|**A**| = 2*5-(-2)*4=10+8=18. $B = \begin{bmatrix} 2 & -2 \\ 4-2*2 & 5-2*(-2) \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 0 & 9 \end{bmatrix}$ |**B**| =2*9-0*(-2)=18

It is a good thing we came out with the same number because I told you that adding a multiple of a row to another row will not change the determinant. The same property holds for adding a multiple of one column to another column.

7) (20 points) Find |**A**| if **A** = $\begin{bmatrix} 2 & 1 & 3 \\ 0 & -1 & 4 \\ -2 & 0 & 5 \end{bmatrix}$

Since you were not asked to get the inverse, we do not need all cofactors. We only need the cofactors we will be using for the determinant. Looking for zeros and ones, the middle column looks best for doing cofactor expansion. So, we need C_{12}, C_{22}, and C_{32}. That requires first getting the minors M_{12}, M_{22}, and M_{32}. $M_{12} = \begin{vmatrix} 0 & 4 \\ -2 & 5 \end{vmatrix}$ = 0 -(-2)*4 = 8. $M_{22} = \begin{vmatrix} 2 & 3 \\ -2 & 5 \end{vmatrix}$ = 2*5-(-2*3) = 10+6 = 16. $M_{31} = \begin{vmatrix} 2 & 3 \\ 0 & 4 \end{vmatrix}$ = 2*4-0*3 = 8. Therefore, $C_{12} = (-1)^{1+2}8$ =-8. Similarly, $C_{22} = (-1)^{2+2}16$ = 16 and $C_{32} = (-1)^{3+2}8$ =-8. So, the cofactor expansion by the middle column gives $a_{12}C_{12} + a_{22}C_{22} + a_{32}C_{32}$ = 1*(-8) + (-1)*16 + 0*(-8) = -8 -16 -0 = -24.

8) (20 points) Find |**A**| if **A** = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 0 & -2 & 1 \end{bmatrix}$

Since you were not asked to get the inverse, we do not need all cofactors. We only need the cofactors we will be using for the determinant. Looking for zeros and ones, the bottom row looks best for doing cofactor expansion. (Actually, the first column looks better, but I did a column in #7, so I am doing the bottom row here.) So, we need C_{31}, C_{32}, and C_{33}. That requires first getting the minors M_{31}, M_{32}, and M_{33}. $M_{31} = \begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix}$ = 2*6 -3*4 = 0. $M_{32} = \begin{vmatrix} 1 & 3 \\ 2 & 6 \end{vmatrix}$ = 1*6-2*3 = 6-6 = 0. $M_{31} = \begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix}$ = 1*4-2*2 = 4-4 = 0. Therefore, $C_{31} = (-1)^{3+1}0 = 0$. Similarly, $C_{32} = (-1)^{3+2}0 = 0$ and $C_{33} = (-1)^{3+3}0 = 0$. So, the cofactor expansion by the bottom row gives $a_{31}C_{31} + a_{32}C_{32} + a_{33}C_{33} =$ 0*0 + (-2)*0 + 1*0 = 0. This is easy to explain, remember the rule I cited in my answer to Question #6? If you add a multiple of a row to another row, you will not change the determinant. Well, take the first row and multiply it by -2 and add it to the second row. The second row becomes all zeros. So, doing the cofactor expansion by that row will give you the sum of 0 times something plus 0 times something plus 0 times something. So, it = 0.